OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754

Applications of Advanced Mathematics (C4)

INSTRUCTIONS

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes + up to 1 hour

The paper is in two parts:

Section A (1 hour 30 minutes)
Section B (up to 1 hour)

Supervisors are requested to ensure that Section B **is not issued** until Section A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Invigilators are not required to match up candidates' two parts. Part A and Part B should be sent to the examiner as two sets of scripts with candidates in the same order as the attendance register for each set.

This notice must be on the Invigilator's desk at all times during the afternoon of Thursday 16 June 2005.

These instructions consist of 1 printed page and 1 blank page.

SP (NF) S90260/1

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS — 4754(A)

Applications of Advanced Mathematics (C4)

Section A

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this section is 72.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

2 Find the first 4 terms in the binomial expansion of $\sqrt{4+2x}$. State the range of values of x for which the expansion is valid. [6]

3 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]

4 Fig. 4 shows a sketch of the region enclosed by the curve $\sqrt{1+e^{-2x}}$, the x-axis, the y-axis and the line $x = 1$.

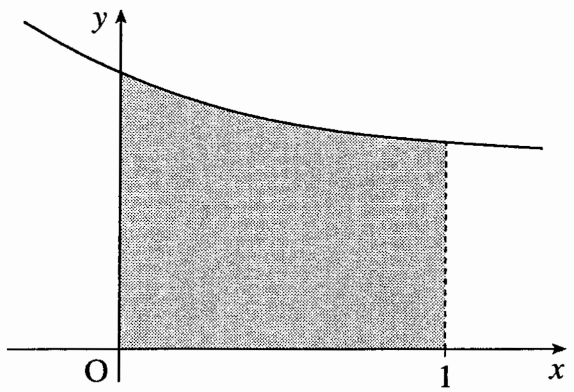

Fig. 4

Find the volume of the solid generated when this region is rotated through 360° about the x-axis. Give your answer in an exact form. [5]

5 Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

6 A curve has cartesian equation $y^2 - x^2 = 4$.

(i) Verify that

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t},$$

are parametric equations of the curve. [2]

(ii) Show that $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t^2+1}$. Hence find the coordinates of the stationary points of the curve. [6]

Section B (36 marks)

7 In a chemical process, the mass M grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dM}{dt} = \frac{M}{t(1+t^2)}.$$

(i) Find $\int \frac{t}{1+t^2}\,dt$. [3]

(ii) Find constants A, B and C such that

$$\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}.$$ [5]

(iii) Use integration, together with your results in parts **(i)** and **(ii)**, to show that

$$M = \frac{Kt}{\sqrt{1+t^2}},$$

where K is a constant. [6]

(iv) When $t = 1, M = 25$. Calculate K.

What is the mass of the chemical in the long term? [4]

8 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the line of the hole.

A 20 cm × 30 cm × 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to x-, y- and z-axes as shown in Fig. 8.1.

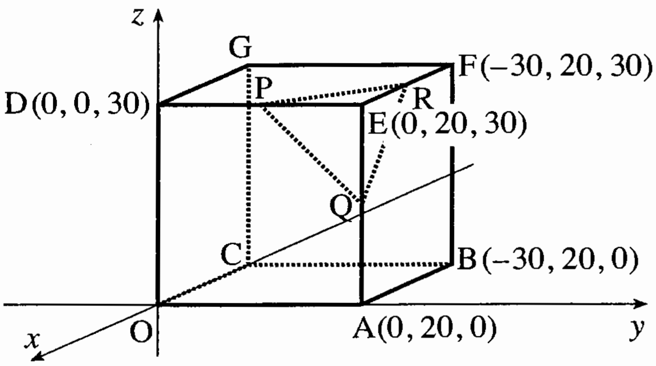

Fig. 8.1

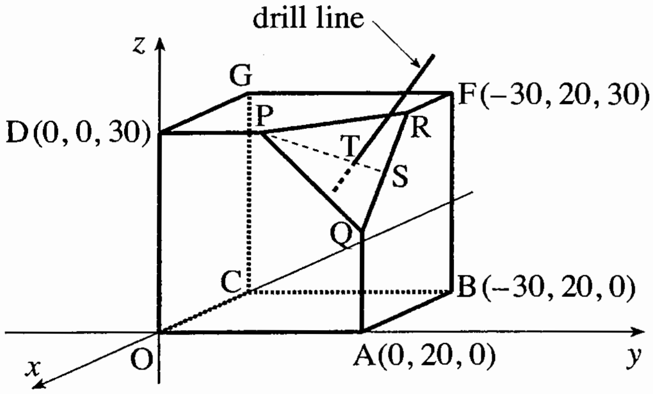

Fig. 8.2

First, a plane cut is made to remove the corner at E. The cut goes through the points P, Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Hence show that $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$. [4]

(ii) Show that the vector $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ is perpendicular to the plane through P, Q and R.

Hence find the cartesian equation of this plane. [5]

A hole is then drilled perpendicular to triangle PQR, as shown in Fig. 8.2. The hole passes through the triangle at the point T which divides the line PS in the ratio 2:1, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates $(-5, 16\frac{2}{3}, 25)$. [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. [5]

Candidate Name	Centre Number	Candidate Number

OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS — 4754(B)

Applications of Advanced Mathematics (C4)

Section B: Comprehension

Thursday **16 JUNE 2005** Afternoon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces at the top of this page.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this section is 18.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
6	
7	
Total	

This question paper consists of 4 printed pages and an insert.

[Turn over

For Examiner's Use

1 Explain why the number 1836.108 for the ratio $\dfrac{\text{Rest mass of proton}}{\text{Rest mass of electron}}$ would be suitable for communication with other civilisations whereas neither the rest mass of the proton nor that of the electron would be. [2]

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2 A civilisation which works in base 5 sends out the first 6 digits of π as 3.032 32. Convert this to base 10. [2]

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3 Complete this table to show the next 3 values of the iteration

$$x_{n+1} = kx_n(1 - x_n)$$

in the case when $k = 3.2$ and $x_0 = 0.5$. Give your answers to calculator accuracy. [1]

n	x_n
0	0.5
1	0.8
2	0.512
3	
4	
5	

For Examiner's Use

4 Justify the statement that the equation in line 83,

$$\frac{\phi}{1} = \frac{1}{\phi - 1},$$

has the solution $\phi = \dfrac{1 \pm \sqrt{5}}{2}$. [2]

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5 Justify the statement in line 87 that

$$\frac{1}{\phi} = \frac{\sqrt{5} - 1}{2}.$$ [3]

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6 A sequence is defined by

$$a_{n+1} = 2a_n + 3a_{n-1} \quad \text{with } a_1 = 1 \text{ and } a_2 = 1.$$

Using the method on page 5, show that the value to which the ratio of successive terms converges is 3.

[4]

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For Examiner's Use

7 Use the information in the article, including the value of Feigenbaum's number given in line 142, to predict an approximate value of k at which the bifurcation from 8 to 16 outcomes occurs for the iterative equation

$$x_{n+1} = kx_n(1 - x_n).$$ [4]

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(B)

Applications of Advanced Mathematics (C4)

Section B: Comprehension

INSERT

Thursday **16 JUNE 2005** Afternoon Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

This insert consists of 8 printed pages.

Communicating with other civilisations

Background

From time immemorial, people have looked into the night sky and wondered whether there are other civilisations out there. During the last hundred years, it has become a theoretical possibility that, if such civilisations do exist, we could communicate with them.

This article looks at some ideas as to how such communication might get started.

First contact

Imagine then that we have reason to think that a planet orbiting a star some light years away might be home to intelligent life. We have no idea what form that life might take, let alone how their society might work.

Natural curiosity means that we would try to make contact with them by sending some sort of message. There would be two requirements.

(i) The message would have to be such that it could be understood anywhere, and so free of any human or Earth-related influence.

(ii) We would want to know whether the message had been received and understood, so it would have to invite a reply.

Many people believe that mathematics is the only universal culture-free language on Earth. So it would be appropriate to use it as a basis for first contact.

One suggestion is that we should transmit the first 5 (say) digits of π, 3.1415. This could be done using short pulses, with a longer one for the decimal point, as illustrated in Fig. 1.

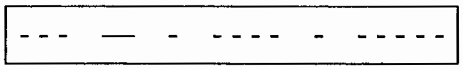

Fig. 1

We would then await a reply consisting of the next 5 digits, 92653.

Will they understand π?

The number we call π arises because it is the ratio of the circumference to the diameter of a circle. It also occurs in many other aspects of mathematics.

An important point is that π requires no units. It is one length divided by another so that, provided the lengths are measured in the same units, the units cancel out. You get the same answer whether you measure the circumference and diameter in metres, feet, miles or anything else. Thus π is a pure number; it is dimensionless.

However, the digits we associate with π are dependent on our number system which uses base 10. If we used base 8, the first 6 digits of π would be 3.11037 since

$$\pi \approx 3 + \frac{1}{8} + \frac{1}{8^2} + \frac{0}{8^3} + \frac{3}{8^4} + \frac{7}{8^5},$$

compared with the base 10 equivalent of

$$\pi \approx 3 + \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{5}{10^4} + \frac{9}{10^5}.$$

The reason we use base 10 is probably because we have a total of 10 fingers and thumbs. So another civilisation might well use a different number base. However, it is perhaps reasonable to assume that, if they are intelligent enough to communicate with us, they also have the sense to try out different number bases.

By communicating in base 10, we are telling the other civilisation that the number 10 is for some reason important in our culture.

A reply comes back

We would not, of course, send π just once. The other civilisation would almost certainly miss it! So we would keep on sending it until either we received a reply or we decided to give up. The nearest star is 4.3 light years away, so the soonest we could possibly get a reply would be 8.6 years. If, however, the star of interest was 50 light years away (not far in astronomical terms), the conversation would be even slower, once every 100 years.

The reply would almost certainly consist of two parts: the answer to our question (92653) and a question of their own. This might perhaps be the signal given in Fig. 2, representing the first 5 digits of e, the base of natural logarithms.

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Fig. 2

Receiving this message would be one of the most exciting events in human history. Undoubtedly it would set off a lively debate about what to send next. It is reasonable to conjecture that three groups would be particularly interested.

- Military personnel would want to assess what the outcome would be if we were to find ourselves at war with the other civilisation.
- Others would want to find out about their culture.
- Scientists would hope to learn from them.

Military questions

The military would find themselves in considerable difficulty. They would want to assess the weapons capability of the other civilisation, and so would need to devise a sequence of different numbers (or questions) each of which related to a different stage of weapons development.

However, sending out such a sequence of numbers would be fraught with danger. The other civilisation might well work out the precise purpose of the questions and so deduce our own

[Turn over

level of military sophistication without giving away any information from their side. It could well be seen as sending out a hostile message with the possibility that the other civilisation would then break off communication.

So it is quite possible that direct military involvement would be seen as just too risky.

Cultural questions

The golden ratio

A number that would certainly be considered is the *golden ratio*. The sides of the rectangle in Fig. 3 are in the ratio 1.618... to 1. This is called the golden ratio and the number 1.618... is denoted by ϕ (the Greek letter phi). Such a rectangle is called a *golden rectangle*.

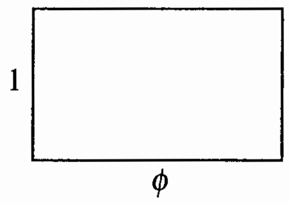

Fig. 3

The golden ratio has considerable cultural significance. For thousands of years a golden rectangle, like the one in Fig. 3, has been regarded as more pleasing to the eye than any other rectangle. So ϕ is related to our artistic sense.

To derive the number ϕ, look at the rectangle ABCD in Fig. 4. It is divided into two parts: a square EBCF at one end, and a smaller rectangle AEFD.

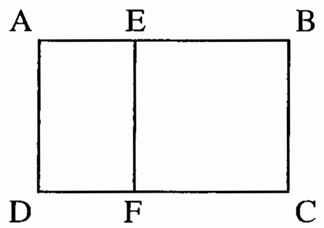

Fig. 4

If the ratio of the sides of the small rectangle AEFD is the same as that of ABCD, then ABCD is a golden rectangle. In this case,

$$\frac{\text{AB}}{\text{AD}} = \frac{\text{AD}}{\text{AE}} = \phi.$$

Thus, if the smaller side, AD, of the main rectangle is given a value of 1 unit, the longer side, AB, is ϕ units.

Since EBCF is a square, the length AE is $(\phi - 1)$ units, and so

$$\frac{\phi}{1} = \frac{1}{\phi - 1}$$

$$\Rightarrow \quad \phi = \frac{1 \pm \sqrt{5}}{2}.$$

Since ϕ must be positive, it follows that the golden ratio is $\frac{1+\sqrt{5}}{2}$ or 1.61803... .

Notice that if the longer side of a golden rectangle is assigned a length of 1 unit, the length of the shorter side is $\frac{1}{\phi} = \frac{\sqrt{5}-1}{2}$ units.

The number $\frac{1+\sqrt{5}}{2}$ crops up in many other places in mathematics, including the Fibonacci sequence. This is usually written

$$1, \; 1, \; 2, \; 3, \; 5, \; 8, \; 13, \; 21, \; 34, \; 55, \; 89, \; \ldots .$$

The first two terms are both given the value 1. After that, each term is the sum of the previous two terms and so the sequence can be defined iteratively by

$$a_{n+1} = a_n + a_{n-1} \quad \text{with } a_1 = 1 \text{ and } a_2 = 1.$$

The ratios of one term to the previous term form the sequence

$$\frac{1}{1} = 1, \; \frac{2}{1} = 2, \; \frac{3}{2} = 1.5, \; \frac{5}{3} = 1.666\ldots, \; \frac{8}{5} = 1.6,$$

$$\frac{13}{8} = 1.625, \; \frac{21}{13} = 1.615\ldots, \; \frac{34}{21} = 1.619\ldots, \; \ldots .$$

This sequence converges and it looks as though its limit has the same value as ϕ.

To prove that it does, imagine that you have taken so many terms that the ratio has settled down to a value, r.

Thus $$r = \frac{a_{n+1}}{a_n} \quad \text{and} \quad r = \frac{a_n}{a_{n-1}}.$$

Take the equation $a_{n+1} = a_n + a_{n-1}$ and divide through by a_n.

$$\frac{a_{n+1}}{a_n} = 1 + \frac{a_{n-1}}{a_n}$$

and so $$r = 1 + \frac{1}{r}$$

$$\Rightarrow \quad r^2 - r - 1 = 0$$

$$r = \frac{1+\sqrt{5}}{2} \quad \left(\text{or } \frac{1-\sqrt{5}}{2}\right).$$

The Fibonacci sequence occurs in nature, for example in connection with the numbers of petals on several types of flowers.

So the message conveyed to another civilisation by the number ϕ would not be unique. They might think we were telling them about our artistic sense, or about the flowers that grow on our planet or about something else.

Feigenbaum's number

A particularly interesting number to send is Feigenbaum's number. This was discovered in 1975 as a result of work on the (then) new subject of chaos.

A simple model for population growth is given by the *logistic equation*,

$$x_{n+1} = kx_n(1 - x_n),$$

where x_n is the population, on a scale of 0 to 1, at a certain time and x_{n+1} is the population one unit of time later. The starting point, x_0, is a number between 0 and 1. The number k represents the reproductivity of the species in question; it is a *parameter* of the model.

This is an iterative process and the outcome depends on the value of k. For $0 \leqslant k \leqslant 1$, the values of x get progressively smaller and converge to zero. The population dies out, whatever the starting value. Table 5(a) illustrates this in the case $k = 0.3$ with starting value $x_0 = 0.5$.

For values of k between 1 and 3, the values of x converge to a particular value which depends on the value of k but not on the starting point. Thus when $k = 2.2$, x converges to 0.545 454 … , as shown in Table 5(b). The population assumes a stable level.

$k = 0.3$	
n	x_n
0	0.5
1	0.075
2	0.020 812 …
3	0.006 113 …
4	0.001 822 …
5	0.000 545 …
6	0.000 163 …
7	0.000 049 …
8	0.000 014 …
9	0.000 004 …
10	0.000 001 …
11	0.000 000 …

Table 5(a)

$k = 2.2$	
n	x_n
0	0.5
1	0.55
2	0.544 5
3	0.545 643 …
4	0.545 416 …
5	0.545 462 …
6	0.545 453 …
7	0.545 454 …
8	0.545 454 …
9	0.545 454 …
10	0.545 454 …
11	0.545 454 …

Table 5(b)

For values of k a little bigger than 3, the value of x ends up oscillating between two outcomes. Thus when $k = 3.2$, the value of x ends up oscillating between 0.513… and 0.799… . The population goes up and down regularly.

For slightly larger values of k, the value of x ends up oscillating between 4 or 8 outcomes.

Fig. 6 shows the outcomes for values of k between 1 and 3.55.

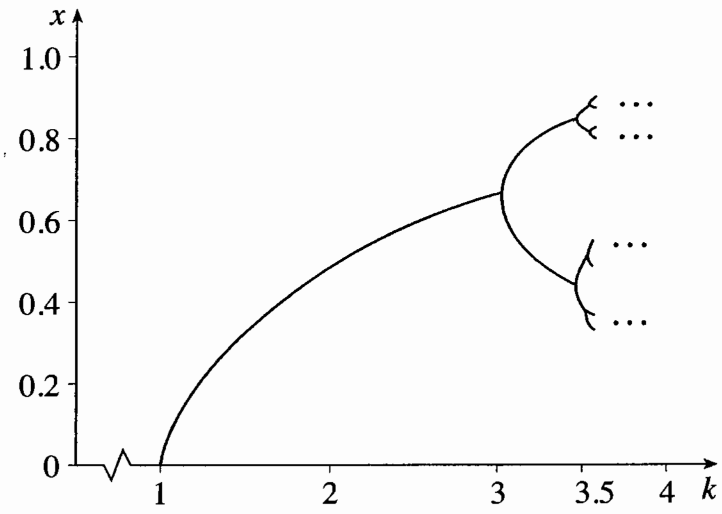

Fig. 6

Larger values of k than those illustrated in Fig. 6 produce even more outcomes, 16, 32 etc. The number of outcomes is always a power of 2.

For still larger values of k there is no pattern at all. The system goes into chaos.

When $k = 3$, there is a change of regime from one non-zero outcome to two. This is called a *point of bifurcation*.

The next point of bifurcation occurs when the number of outcomes changes from 2 to 4; it occurs at $k = 3.4485$ (to 4 decimal places). The one after that is at $k = 3.5437$ when the change is from 4 to 8, and so on.

Number of non-zero outcomes	**1**	**2**	**4**
Values of k	1 to 3	3 to 3.4485	3.4485 to 3.5437
Length of present interval	2	0.4485	0.0952
$\dfrac{\textbf{Previous interval}}{\textbf{Present interval}}$	–	$\dfrac{2}{0.4485} = 4.459\ldots$	$\dfrac{0.4485}{0.0952} = 4.711\ldots$

Table 7

Table 7 shows the intervals, in terms of the parameter k, for the first 3 regimes (excluding $0 \leqslant k \leqslant 1$ for which the population dies out). The final row gives the ratios of lengths of consecutive intervals.

What Feigenbaum discovered is that the sequence formed by the values of this ratio converges to a particular number, 4.669 201… . (Australian mathematicians have now computed it to 1000 significant figures.)

What is remarkable about Feigenbaum's number is that it arises not just in the iterative equation described above, but in a wide variety of equations modelling real-life situations which change from order into chaos. It is a universal constant.

For example, an entirely different iterative equation which generates Feigenbaum's number is

$$x_{n+1} = k\sin(\pi x_n).$$

The fact that 30 years ago we did not know about Feigenbaum's number, but do now, illustrates the point that it is a marker in our technological and cultural development. Its discovery depended on having the power of electronic calculation. The number and length of the calculations involved in its discovery were enormous.

So sending the first few digits of Feigenbaum's number to another civilisation could be taken as asking the question "Have you developed computers yet?" However, a positive response could also mean that those in the other civilisation have wonderfully good brains of their own, so good that they do not need computers.

Scientific questions

It would be a matter of great interest to scientists to find out whether quantities we believe to be constant really are: for example the velocity of light, $2.997\,924 \times 10^5$ kilometres per second. Even a small difference in this would require a fundamental re-think of the laws of physics.

Unfortunately this value uses units that are derived from conditions on Earth. The kilometre is approximately $\frac{1}{40\,000}$ of the circumference of the Earth, and the second is approximately $\frac{1}{60 \times 60 \times 24}$ of the time it takes the Earth to spin once on its axis (i.e. one day). Although both of these units are now defined more precisely using basic properties of matter, they retain essentially the same values and so would be meaningless to another civilisation.

It may be that the best that scientists could achieve would be certain ratios. For example

$$\frac{\text{Rest mass of proton}}{\text{Rest mass of electron}} = 1836.108.$$

The numbers scientists would send would almost certainly have been determined experimentally, to a known level of accuracy. If the reply came back with the same number but given to a much higher level of accuracy, it would be a good indication that the other civilisation is more technologically advanced than we are.

Thus it may be that the answers to science-based questions would tell us more about the civilisation's level of development than about science.

Conclusion

There are many other numbers that could be used in this context. No doubt a selection committee would be needed! The numbers mentioned in this article illustrate some of the principles which might guide the work of such a committee.

Mark Scheme 4754
June 2005

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓
   - For error in follow through work, use ✓

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not been earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

c.a.o.	: correct answer only
b.o.d.	: benefit of doubt (where full work is not shown)
X ... X	: work of no mark value between crosses
s.o.i.	: seen or implied
s.c.	: special case (as defined in the mark scheme)
w.w.w	: without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting prior to the batch 1 date, send a further sample of about 40 scripts, from complete centres. You should record the marks for these scripts on your marksheets. They will not be returned to you, but you will receive feedback on them. If all is well, you will then be given clearance to send your batch 1 scripts and marksheets to Cambridge.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

SECTION A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25,\ R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.927$ $f(\theta) = 7 + 5\cos(\theta - 0.927)$ $\Rightarrow$ Range is 2 to 12 Greatest value of $\dfrac{1}{7+3\cos\theta+4\sin\theta}$ is ½ .	**B1** **M1** **A1** **M1** **A1** **B1ft** **[6]**	R=5 tan α=4/3 oe ft their R 0.93 or 53.1° or better their cos (θ - 0. 927) = 1 or -1 used (*condone use of graphical calculator)* 2 and 12 seen cao simplified
2 $\sqrt{4+2x} = 2(1+\frac{1}{2}x)^{\frac{1}{2}}$ $= 2\{1+\frac{1}{2}.(\frac{1}{2}x)+\frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(\frac{1}{2}x)^2+\frac{\frac{1}{2}.(-\frac{1}{2}).(-\frac{3}{2})}{3!}(\frac{1}{2}x)^3+...\}$ $= k\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3+...\right)$ $= \left(2+\frac{1}{2}x-\frac{1}{16}x^2+\frac{1}{64}x^3+...\right)$ Valid for $-2 < x < 2$.	**M1** **M1** **A2,1,0** **A1cao** **B1cao** **[6]**	Taking out 4 oe correct binomial coefficients $\frac{1}{4}x, -\frac{1}{32}x^2, +\frac{1}{128}x^3$
3 $\sec^2\theta = 4$ $\Rightarrow \dfrac{1}{\cos^2\theta} = 4$ $\Rightarrow \cos^2\theta = ¼$ $\Rightarrow \cos\theta = ½$ or $-½$ $\Rightarrow \theta = \pi/3, 2\pi/3$ **OR** $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow \tan^2\theta = 3$ $\Rightarrow \tan\theta = \sqrt{3}$ or $-\sqrt{3}$ $\Rightarrow \theta = \pi/3, 2\pi/3$	**M1** **M1** **A1 A1** **M1** **M1** **A1 A1** **[4]**	sec θ = 1/cos θ used ± ½ allow unsupported answers ± √3 allow unsupported answers

4 $V = \int \pi y^2 dx$ $= \int_0^1 \pi(1+e^{-2x})dx$ $= \pi\left[x - \frac{1}{2}e^{-2x}\right]_0^1$ $= \pi(1 - ½e^{-2} + ½)$ $= \pi(1½ - ½e^{-2})$	**M1** **M1** **B1** **M1** **A1** **[5]**	Correct formula $k\int_0^1 (1+e^{-2x})dx$ $\left[x - \frac{1}{2}e^{-2x}\right]$ substituting limits. Must see *0* used. Condone omission of π o.e. but must be exact
5 $2\cos 2x = 2(2\cos^2 x - 1) = 4\cos^2 x - 2$ $\Rightarrow 4\cos^2 x - 2 = 1 + \cos x$ $\Rightarrow 4\cos^2 x - \cos x - 3 = 0$ $\Rightarrow (4\cos x + 3)(\cos x - 1) = 0$ $\Rightarrow \cos x = -3/4$ or 1 $\Rightarrow x = 138.6°$ or $221.4°$ or 0	**M1** **M1** **M1dep** **A1** **B1 B1** **B1** **[7]**	Any double angle formula used *getting a quadratic in cos x* attempt to solve for -3/4 and 1 139,221 or better www -1 extra solutions in range
6 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$ $= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$ $= 4$	**M1** **E1** **[2]**	Substituting for x and y in terms of t oe
(ii) EITHER $dx/dt = 1 + 1/t^2$, $dy/dt = 1 - 1/t^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1-1/t^2}{1+1/t^2}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ * **OR** $2y\frac{dy}{dx} - 2x = 0$ $\Rightarrow \frac{dy}{dx} = \frac{x}{y} = \frac{t-1/t}{t+1/t}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ **OR** $y = \sqrt{(4+x^2)}$, $\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{4+x^2}}$ $= \frac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$ $= \frac{t-1/t}{\sqrt{(t+1/t^2)}} = \frac{t-1/t}{(t+1/t)}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ $\Rightarrow dy/dx = 0$ when $t = 1$ or -1 $t = 1, \Rightarrow (0, 2)$ $t = -1 \Rightarrow (0, -2)$	**B1** **M1** **E1** **B1** **M1** **E1** **B1** **M1** **E1** **M1** **A1 A1** **[6]**	For both results

SECTION B

7 (i) $\int \frac{t}{1+t^2}dt = ½\ln(1+t^2)+c$ **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, $du = 2t\,dt$ $= \int \frac{1/2}{u}du$ $= ½ \ln u + c$ $= ½ \ln(1 + t^2) + c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k\ln(1+t^2)$ $½\ln(1+t^2)$ [$+c$] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow$ $1 = A(1+t^2) + (Bt + C)t$ $t = 0 \Rightarrow 1 = A$ coeff^t of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff^t of t $\Rightarrow 0 = C$ $\Rightarrow$ $\frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	 **M1** **M1** **A1** **A1** **A1** **[5]**	 Equating numerators substituting or equating coeff^{t}s dep 1st M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow$ $\int \frac{1}{M}dM = \int \frac{1}{t(1+t^2)}dt = \int [\frac{1}{t} - \frac{t}{1+t^2}]dt$ $\Rightarrow$ $\ln M = \ln t - ½\ln(1+t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow$ $M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	**M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½\ln(1+t^2) + c$ combining $\ln t$ and $½\ln(1+t^2)$ $K = e^c$ o.e.
(iv) $t = 1, M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow$ $K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.
8 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (−15, 20, 30) $\Rightarrow$ $\overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow$ $\overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$	**M1**	Scalar product with 1 vector in the plane OR vector x product oe
$\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$	**E1**	
$\Rightarrow$ $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane		
$\Rightarrow$ equation of plane is $2x + 3y + 2z = c$	**M1**	$2x + 3y + 2z = c$ **or** an appropriate vector form
At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow$ $c = 2\times0 + 3\times10 + 2\times30 = 90$	**M1dep**	substituting to find c **or** completely eliminating parameters
$\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**A1 cao** **[5]**	
(iii) S is $(-7\frac{1}{2},20,22\frac{1}{2})$	**B1**	
$\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$	**M1**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S
$=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$	**A1ft**	**Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
So T is $(-5,16\frac{2}{3},25)$ *	**E1** **[4]**	
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$	**B1,B1**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\ldots\ \ldots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$
At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$	**M1** **A1**	Substituting coordinates of C into vector equation. At least 2 relevant correct equations for λ
1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**E1** **[5]**	oe www

COMPREHENSION

1. The masses are measured in units. The ratio is dimensionless	**B1** **B1** **[2]**	
2. Converting from base 5, $3.03232 = 3+\frac{0}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\frac{2}{5^5}$ $= 3.14144$	**M1** **A1** **[2]**	
3. <table><tr><th>n</th><th>x_n</th></tr><tr><td>0</td><td>0.5</td></tr><tr><td>1</td><td>0.8</td></tr><tr><td>2</td><td>0.512</td></tr><tr><td>3</td><td>**0.7995392**</td></tr><tr><td>4</td><td>**0.5128840565**</td></tr><tr><td>5</td><td>**0.7994688035**</td></tr></table>	**B1**	Condone variations in last digits
4. $\frac{\phi}{1}=\frac{1}{\phi-1}$ $\Rightarrow \phi^2-\phi=1 \quad \Rightarrow \phi^2-\phi-1=0$ Using the quadratic formula gives $\phi=\frac{1\pm\sqrt{5}}{2}$	**M1** **E1**	Or complete verification B2
5. $\frac{1}{\phi}=\frac{1}{\frac{1+\sqrt{5}}{2}}=\frac{2}{1+\sqrt{5}}$ $=\frac{2}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$ $=\frac{2(\sqrt{5}-1)}{(\sqrt{5})^2-1}=\frac{2(\sqrt{5}-1)}{4}=\frac{\sqrt{5}-1}{2}$ **OR** $\frac{1}{\phi}=\phi-1$ $=\frac{\sqrt{5}+1}{2}-1=\frac{\sqrt{5}-1}{2}$	**M1** **M1** **E1** **M1** **M1** **E1** **[3]**	Must discount ± Must discount ± Substituting for ϕ and simplifying

6. Let $r = \dfrac{a_{n+1}}{a_n}$ and $r = \dfrac{a_n}{a_{n-1}}$	**M1**	For either ratio used
$a_{n+1} = 2a_n + 3a_{n-1}$		
dividing through by a_n. $\Rightarrow r = 2 + \dfrac{3}{r}$	**M1**	$r = 2 + \dfrac{3}{r}$
$\Rightarrow r^2 - 2r - 3 = 0$ $\Rightarrow (r-3)(r+1) = 0$	**A1**	
$\Rightarrow$ $r = 3$ (discounting -1)	**E1** **[4]**	SC B2 if dividing terms as far as $a_9/a_8 = a_{10}/a_9$ =3.00
7. The length of the next interval = l, where $\dfrac{0.0952...}{l} = 4.669...$	**M1**	
$\Rightarrow$ $l = 0.0203$	**A1**	
So next bifurcation at 3.5437… + 0.0203…≈ 3.564	**DM1** **A1** **[4]**	

4754 - Applications of Advanced Mathematics

General Comments

This was the first time that this examination has been set. The examination for this new specification was longer than the previous one with more questions and the Comprehension was slightly longer.

The standard of work was quite good although there was a wide variety between centres. Some achieved an excellent standard with high scores and well reasoned solutions but, equally, there was some disappointing work from other centres both in content and presentation.
The Comprehension was the least successfully answered question. It scored, on average, approximately one third of the marks available and generally had a negative overall effect on the candidates' scores.
Candidates should be encouraged to:

- consider coefficients when using partial fractions
- use a logarithmic constant when integrating when all other terms are logarithms
- show fully the scalar product of the normal vector with two vectors in the plane to show that it is, in fact, perpendicular to the plane
- use the approach from $ax+by+cz=d$ when finding the Cartesian equation of a plane.
- realise that they should attempt the Comprehension questions as they represent a significant proportion of the marks.

Comments on Individual Questions

1) The first three marks were usually scored. Almost all candidates successfully found that R=5. The angle was usually found correctly but the answer was often given in degrees rather than radians. The second part of the question was less successful. Most candidates failed to realise that they were expected to use the fact that cos x must vary between -1 and +1 and either missed this part out or tried to solve an equation such as $f(\theta)=0$. Good candidates were, however, successful here.

2) There were some good answers for the binomial expansion. The binomial coefficients were usually correct. Many candidates found it difficult to factorise the 4 out of the bracket successfully. They often used a factor of 4 or ½ instead of $\sqrt{4}$ or 2. Further errors often arose as a result of brackets being missing around the terms in ($\frac{1}{2}x$).There were also errors in cancelling fractions.
Many candidates-including many good candidates- failed to give the range for which the expansion was valid. Others gave the answer as $-2\leq x\leq 2$ instead of inequalities or answered $-1/2 < x < 1/2$.

3) Appropriate trigonometric identities were usually used but the majority of candidates failed to obtain the negative root and hence missed the second solution of $2\pi/3$. Some failed to give their solutions in terms of π as required. Most candidates scored three out of the possible four marks available.

4) The majority of candidates used the correct formula here. Most scored well. The most common error in the integration was to multiply by 2 instead of dividing when integrating the exponential. A disappointing number of candidates failed to substitute the lower zero limit into the integrand. Another frequent error was failing to leave the answer in its exact form.

5) This question saw candidates scoring both the highest number of zero marks and the highest number of full marks. Candidates who did not attempt to use a double angle substitution for cos $2x$ were generally unable to proceed. Many tried, incorrectly, to use cos $2x=\cos^2 x -1$. Some made an incorrect substitution but understood the general method. They therefore achieved some marks by forming a quadratic equation and attempting to solve it. In general further marks were rarely scored except by those that chose a relevant initial substitution. For those candidates that started correctly full marks were usually scored. Occasionally additional incorrect solutions were given.

6) (i) Although a few candidates failed to attempt the first part, the majority of those that did achieved at least the first method mark for substituting for y and x in terms of t into $y^2-x^2=4$. The most common error being incorrect expansions of $\left(t+1/t\right)^2$ and $\left(t-1/t\right)^2$ -usually omitting the middle term.

(ii) This part was approached in a variety of different ways. The most common involved finding $\mathrm{d}y/\mathrm{d}t$ and $\mathrm{d}x/\mathrm{d}t$ and dividing. Although the method was generally understood there were errors in simplification to the given result. The most common error was $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-1/t^2}{1+1/t^2} = \frac{1-t^2}{1+t^2}.$

Other errors included the incorrect use of $\mathrm{d}x/\mathrm{d}t=1+1/t^2$ so $\mathrm{d}t/\mathrm{d}x=1+t^2$.

Similar errors occurred when the question was approached via implicit differentiation. Using the explicit differentiation of $y=\sqrt{(4+x^2)}$ and substitution was seen less often.

The final part of the question was missed out many times with the loss of some relatively easy marks. Of those that did attempt it there were too many simple arithmetical errors often leading to only one set of co-ordinates being correct. Many did, however, complete this successfully.

7) (i) Often fully correct but some failed to realise this was a logarithm. Most candidates obtained the answer directly rather than by substitution.

(ii) Many good marks were scored here. Some candidates failed to include brackets around the $Bt+C$ term. There were some long solutions involving substitutions whereas comparing coefficients was quicker and less prone to error.

(iii) The first step here should have been to separate the variables. Several candidates retained the *M* on the right hand side of the equation as if it were a constant. Other candidates omitted the *M*s and only proceeded with the right hand side. The substitution of partial fractions was usually correct as was the following integration but many candidates omitted the constant at this stage. Although candidates were usually able to demonstrate that they knew how to combine logarithms successfully, there were very few instances where the constant was dealt with correctly. Using a logarithmic constant should be encouraged in such questions as fewer errors are incurred with this approach. There were a few excellent complete solutions to this part but they were rare. Weaker candidates often omitted this integration.

(iv) Nearly all found *K* correctly and many obtained full marks but *M* tending to 0 or infinity was a fairly common answer.

8) (i) Usually high scoring and well answered.

(ii) Candidates should be reminded that it is necessary to show a vector is perpendicular to **two** independent vectors in a plane in order to establish that it is perpendicular to the plane. One is not sufficient.
Too few candidates showed the numerical evaluation of their scalar products resulting in the unnecessary loss of marks.
When finding the Cartesian equation of a plane the candidates should be advised to use the approach via $2x+3y+2z=$ *a constant.* Those that started using a vector equation and eliminating parameters made more errors and their method took longer.

(iii) The co-ordinates were often found correctly. Although some candidates did give good clear solutions in finding the co-ordinates of T, many gave unclear, confused solutions with poor notation and reasoning that was difficult to follow. As the answer was given in the question there was a tendency to try to acquire the correct solution from incorrect figures.

(iv) The vector equation of the line of the drill hole was usually given correctly although some candidates confused the position vector and the direction of the line. Both vectors were given in the question but some candidates still made errors.
Thereafter, those that made a serious attempt produced reasonable answers but there were numerical errors in the final stage. The majority chose to show that the point C did not lie on the vector equation of the line although a few gave an equally reasoned argument that the direction of the vector **CT** was not in the same direction as that of the drill hole.

Section B: Comprehension

1) Candidates were required to give an answer that showed an understanding of the fact that the units we use would not be understood in other civilisations but that ratios which are dimensionless would be understood everywhere. There were some good solutions but many others missed the point.
Some said that mass would be different on other planets or that gravity would be different. Others wrote of the intelligence of other civilisations.

2) The number bases proved difficult. Many candidates correctly obtained the method mark by stating $3.03232=3+0/5+3/5^2+2/5^3+3/5^4+2/5^5$ or equivalent but then failed to obtain the answer mark-often quoting 3.14159 instead.

3) This was the mark that was most frequently scored in the comprehension. Some candidates failed to give the requested full calculator display.

4) A large number of candidates found the quadratic equation and hence the solution. In a number of cases the solution was just stated rather than being established and then the E mark could not be given as the answer was given in the question.
Several candidates tried to use a numerical value of ϕ.

5) There were several methods of approaching this but good solutions were rare. Candidates needed to discount the negative root and then simplify their expression.

6) There were a few very good solutions here but some tried to use an incorrect expression for r such as $r = \dfrac{a_{n+1}}{2a_n}$ and $r = \dfrac{2a_n}{3a_{n-1}}$.

Many others found successive terms and their ratios but either made numerical errors when calculating the terms or did not calculate as far as the a_{10}/a_9 ratio or both. Others did not attempt the question.

7) This was not attempted by many. For those with the right approach there were good solutions. In general either 0 or 4 marks were scored. Several candidates found the required value of *k* and, unnecessarily, the subsequent one too.